

Polarimetry in the human eye using an imaging linear polariscope

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Abstract

We have studied the polarization parameters of the human eye associated with ocular birefringence from double-pass retinal images by using an imaging linear polariscope. A series of nine images corresponding to combinations of linear independent polarization states in both generator and analyser units was recorded. Retardation and azimuthal angle obtained when considering the human eye as a linear retarder have been compared to those calculated with a Mueller matrix polarimeter. Results in young eyes show only small differences, of about 2° for azimuth and 6° for retardation, between these methods. Moreover, changes in the polarization state of the central part of double-pass images are very different from those corresponding to the tails. Although the simpler linear polariscope is mainly designed for studies in physiological optics and clinical diagnosis, it can also be used for the analysis of *in vitro* biological samples and crystals.

Keywords: Retardation, azimuth, Mueller matrix, polariscope

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Stokes–Mueller formalism describes the polarization properties of light beams and samples. In terms of polarization, a light beam with intensity I is expressed by means of a 4×1 column vector called a Stokes vector, $S = [S_0, S_1, S_2, S_3]^T$ with $S_0 = I$ [1]. The elements of S satisfy the relationship $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$. If the light beam is totally polarized $S_0^2 = S_1^2 + S_2^2 + S_3^2$, and $S_i = 0$ ($i = 1, 2, 3$) when the light is depolarized. Otherwise, the light will be partially polarized. The degree of polarization (DOP) of a light beam is defined as the ratio of the polarized-component intensity to the total intensity.

The polarization properties of a system are described by a 4×4 matrix, termed the Mueller matrix, M , which transforms incident Stokes vectors into exiting Stokes vectors. These properties depend on both internal and external structures and are classified as diattenuation, birefringence, depolarization and polarizance (see, for instance, [1] for further information).

A polarimeter permits the measurement of Stokes vectors and Mueller matrices by using a generator and an analyser, both producing (four) independent polarization states and

composed of a fixed linear polarizer and a compensator (quarter-wave plate, variable retarder) [2–5]. When the full Mueller matrix or the complete Stokes vector cannot be reconstructed, the experimental system is known as an incomplete polarimeter [1] or polariscope [6] (or as a linear polariscope when incorporating just linear polarizers). These set-ups are often used when one polarization property of the system is much more important than the rest and the calculation of the complete Mueller matrix is not required [2, 7–9].

The ocular media and the retina are an example containing the above polarization properties (see [10] as a general reference). Experimental systems combining imaging and polarization have been successfully used to study spatially resolved polarization properties in the living human eye [3, 11–13]. The analysis of light which is reflected by the retina gives information about these ocular properties which have been widely used in clinical diagnosis and the detection of pathologies [14–20]. However, many previous experiments did not use a polarimeter, but rather a linear polariscope. These studies interpreted the results just in terms of two components: one maintaining the polarization and the other

becoming depolarized [21–23]. This kind of set-up can neither generate circular polarized light nor determine the circular polarization content of a beam. As a consequence, the whole Mueller matrix (and alternatively the four elements of the Stokes vector) cannot be computed.

Despite linear birefringence having been reported to be the most important polarization property in the human eye [12, 24, 25], studies centred on whether to consider the eye as a fairly good linear retarder is lacking in the literature. In this paper, we consider the pros and cons of using a simple linear polariscope. This polarimetric configuration has been applied to the study of polarization properties (associated with birefringent structures) of young healthy eyes. An analysis of the effect of a rotating analyser in retinal double-pass images is also presented.

This paper also includes the calculation of nine elements of the Mueller matrix as well as a simple method for extracting the retardation and azimuthal angles of highly birefringent samples when using a linear polariscope. Finally, the appendix shows, in an easy way, how totally elliptically polarized light can be identified as partially polarized when using a linear polarizer as analyser. Throughout this work we will refer to a complete polarimeter (polarizer and compensator in both generator and analyser units) as MM. A polariscope will be termed PA when having rotating linear polarizers in both generator and analyser units. If the generator is fixed, it will be called FPA.

2. Polarimetry using a linear polariscope: theory

2.1. Calculation of the elements of the Mueller matrix

When working with a polariscope (FPA or PA) the number of intensity measurements is noticeably reduced [6]. However, the accuracy and performance of these polarimetric devices depend on whether the effects due to birefringence of the optical system are clearly dominant (diattenuation and depolarization are much lower) or not. On the other hand, the use of a PA configuration allows the calculation of nine elements of the Mueller matrix, which does not imply any hypothesis about the optical properties of the system under study. In the following this will be explained.

Let us suppose a PA configuration. Three different orientations of the transmission axis (45° apart) of the polarizer used as a generator produce three independent linearly polarized states: horizontal ($S_{IN}^{(1)}$), vertical ($S_{IN}^{(2)}$) and at 45° ($S_{IN}^{(3)}$). If these states enter the sample under study, represented by a Mueller matrix $M = m_{kl}$ ($k, l = 0, 1, 2, 3$), the emergent states $\tilde{S}_{OUT}^{(i)}$ ($i = 1, 2, 3$) will be given by

$$\begin{aligned} \begin{pmatrix} m_{00} + m_{01} \\ m_{10} + m_{11} \\ m_{20} + m_{21} \\ m_{30} + m_{31} \end{pmatrix} &= M \cdot S_{IN}^{(1)} = \tilde{S}_{OUT}^{(1)}; \\ \begin{pmatrix} m_{00} - m_{01} \\ m_{10} - m_{11} \\ m_{20} - m_{21} \\ m_{30} - m_{31} \end{pmatrix} &= M \cdot S_{IN}^{(2)} = \tilde{S}_{OUT}^{(2)}; \\ \begin{pmatrix} m_{00} + m_{02} \\ m_{10} + m_{12} \\ m_{20} + m_{22} \\ m_{30} + m_{32} \end{pmatrix} &= M \cdot S_{IN}^{(3)} = \tilde{S}_{OUT}^{(3)}. \end{aligned} \quad (1)$$

Let $S_{OUT}^{(i)}$ be the 3×1 vectors without including the fourth element of $\tilde{S}_{OUT}^{(i)}$. If M_{PSA} is the auxiliary 3×3 matrix with each row being the first row of every Mueller matrix corresponding to a different orientation of the transmission axis of the analyser (0° , 90° and 45°) without the last element, then it is verified that

$$\begin{pmatrix} I_1^{(i)} \\ I_2^{(i)} \\ I_3^{(i)} \end{pmatrix} = M_{PSA} \cdot S_{OUT}^{(i)} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot S_{OUT}^{(i)} \quad (2)$$

where $I_j^{(i)}$ ($j = 1, 2, 3$) are the registered intensities for each orientation of the analyser and a fixed incoming polarization state $S_{IN}^{(i)}$. When using a rotating linear polarizer as analyser, just three components of the Stokes vector $S_{OUT}^{(i)}$ can be calculated by inversion of equation (2):

$$\begin{aligned} S_{OUT}^{(i)} &= \begin{pmatrix} S_0^{(i)} \\ S_1^{(i)} \\ S_2^{(i)} \end{pmatrix} = (M_{PSA})^{-1} \begin{pmatrix} I_1^{(i)} \\ I_2^{(i)} \\ I_3^{(i)} \end{pmatrix} \\ &= \begin{pmatrix} I_1^{(i)} + I_2^{(i)} \\ I_1^{(i)} - I_2^{(i)} \\ -I_1^{(i)} - I_2^{(i)} + I_3^{(i)} \end{pmatrix}. \end{aligned} \quad (3)$$

This procedure can be applied to any of the three incident vectors $S_{IN}^{(i)}$ and nine elements of the Mueller matrix are easily obtained by means of

$$\begin{pmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{20} \\ m_{21} \\ m_{22} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & -1/2 & -1/2 & 0 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & -1/2 & -1/2 & 0 & 1 & 1 & 0 \\ 1/2 & -1/2 & 0 & 1/2 & -1/2 & 0 & 0 & 0 & 0 \\ 1/2 & -1/2 & 0 & -1/2 & 1/2 & 0 & 0 & 0 & 0 \\ -1/2 & 1/2 & 0 & -1/2 & 1/2 & 0 & 1 & -1 & 0 \\ -1/2 & -1/2 & 1 & -1/2 & -1/2 & 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 1 & 1/2 & 1/2 & -1 & 0 & 0 & 0 \\ 1/2 & 1/2 & -1 & 1/2 & 1/2 & -1 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} I_1^{(1)} \\ I_2^{(1)} \\ I_3^{(1)} \\ I_1^{(2)} \\ I_2^{(2)} \\ I_3^{(2)} \\ I_1^{(3)} \\ I_2^{(3)} \\ I_3^{(3)} \end{pmatrix}. \quad (4)$$

The full 16 elements of the Mueller matrix are required to compute all the polarization properties of the system. However, some properties, such as linear diattenuation (D_L) and linear polarizance (P_L), can be calculated using the elements shown in equation (4):

$$D_L = \frac{\sqrt{m_{01}^2 + m_{02}^2}}{m_{00}} \quad P_L = \frac{\sqrt{m_{10}^2 + m_{20}^2}}{m_{00}}. \quad (5)$$

D_L characterizes the variation of intensity transmittance with incident linear polarization states. P_L informs us about the DOLP of the transmitted light when depolarized light is incident. Moreover the axis for diattenuation (angle of the axis with maximum transmittance), α , as well as the intensity transmission coefficients (p_1^2 and p_2^2), can also be computed:

$$\begin{aligned} \alpha &= \frac{1}{2} a \tan\left(\frac{m_{02}}{m_{01}}\right) & p_1^2 &= m_{00} + \frac{m_{10}}{\cos(2\alpha)} \\ p_2^2 &= m_{00} - \frac{m_{10}}{\cos(2\alpha)}. \end{aligned} \quad (6)$$

2.2. Derivation of the retardation and the azimuth for a birefringent sample

If diattenuation and depolarization are negligible compare to linear birefringence, the Mueller matrix of the optical system, M_{Δ}^{β} , basically corresponds to a retarder with retardation Δ and azimuth β (fast axis) which is given by [26]

$$M_{\Delta}^{\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2k & sc(1-k) & -sx \\ 0 & sc(1-k) & s^2 + c^2k & cx \\ 0 & sx & -cx & k \end{pmatrix} \quad (7)$$

where $c = \cos 2\beta$, $s = \sin 2\beta$, $k = \cos \Delta$ and $x = \sin \Delta$.

For each incident polarization state, $S_{IN}^{(i)}$ ($i = 1, 2, 3$), the Stokes vectors emerging from this sample are

$$\begin{aligned} \tilde{S}_{OUT}^{(1)} &= \begin{pmatrix} 1 \\ c^2 + s^2k \\ sc(1-k) \\ sx \end{pmatrix} & \tilde{S}_{OUT}^{(2)} &= \begin{pmatrix} 1 \\ -c^2 - s^2k \\ sc(k-1) \\ -sx \end{pmatrix} \\ \tilde{S}_{OUT}^{(3)} &= \begin{pmatrix} 1 \\ sc(1-k) \\ s^2 + c^2k \\ -cx \end{pmatrix}. \end{aligned} \quad (8)$$

If a horizontal linearly polarized light beam ($S_{IN}^{(1)}$) is incident, an intensity can be registered for each orientation of the analyser. With these intensities and equation (3), the corresponding 3×1 emergent vector $S_{OUT}^{(1)} = [S_0^1, S_1^1, S_2^1]^T$ is calculated. Normalizing this vector and operating with the expressions of $S_1^{(1)}$ and $S_2^{(1)}$, the polarization parameters (β and Δ) of the birefringent sample are obtained from

$$\beta = \frac{1}{2}a \tan\left(\frac{1 - S_1^{(1)}}{S_2^{(1)}}\right) \quad \Delta = a \cos\left(1 - \frac{2S_2^{(1)}}{\sin(4\beta)}\right). \quad (9)$$

In a similar way, when $S_{IN}^{(2)}$ or $S_{IN}^{(3)}$ enter the sample, the retardation and azimuthal angles are given by

$$\beta = \frac{1}{2}a \tan\left(-\frac{1 + S_1^{(2)}}{S_2^{(2)}}\right) \quad \Delta = a \cos\left(1 + \frac{2S_2^{(2)}}{\sin(4\beta)}\right) \quad (10)$$

$$\beta = \frac{1}{2}a \tan\left(\frac{S_1^{(3)}}{1 - S_2^{(3)}}\right) \quad \Delta = a \cos\left(1 - \frac{2S_1^{(3)}}{\sin(4\beta)}\right).$$

Example of implementation: a retardation plate. As an example a quarter-wave plate was placed between two linear polarizers (nominal azimuth 15°). A laser beam (2 mm in diameter) was incident on its central area. A detector (not sensitive to polarization) registered the nine values of intensity, each corresponding to an independent orientation of the generator-analyser. Using these intensities and equations (3), (9) and (10) azimuth and retardation were computed. Results are shown in table 1.

The elements of the Mueller matrix have been calculated using equation (4):

$$\begin{pmatrix} 1 & 0.002 \pm 0.001 & 0.010 \pm 0.002 & m_{03} \\ 0.001 \pm 0.004 & 0.745 \pm 0.009 & 0.435 \pm 0.010 & m_{13} \\ 0.008 \pm 0.003 & 0.445 \pm 0.008 & 0.243 \pm 0.011 & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix}.$$

Table 1. Retardation and azimuth (in degrees) of a quarter-wave plate with its fast axis at 15° calculated by using a linear polariscope and equations (3), (9) and (10).

	$S_{IN}^{(1)}$	$S_{IN}^{(2)}$	$S_{IN}^{(3)}$
Retardation	91.80 ± 0.84	91.07 ± 0.42	90.67 ± 0.76
Azimuth	14.95 ± 0.31	15.43 ± 0.21	15.18 ± 0.16

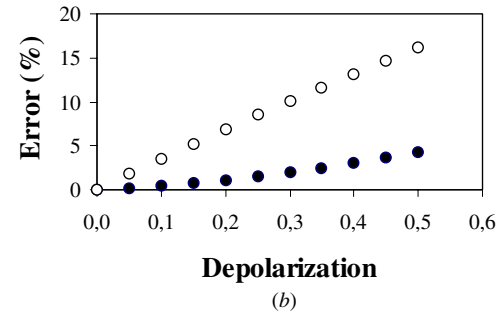
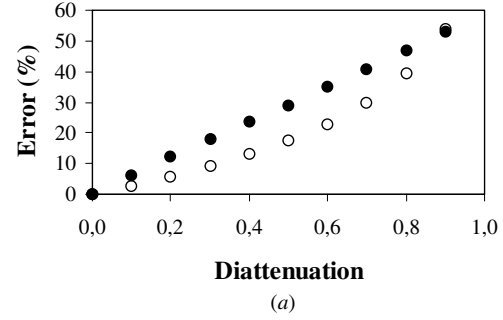


Figure 1. Errors introduced in the calculation of the retardation (Δ) and azimuth (β) of a birefringent sample with different amounts of diattenuation (a) and depolarization (b). Full and open circles correspond to retardation and azimuth respectively.

Using the elements of this matrix $D_L = 0.010$ and $P_L = 0.008$, which indicates that dichroic properties are negligible (as expected) and much smaller than the effects of birefringence.

As a test to check the accuracy of our experiment, we used the above matrix to compute the polarization parameters of the retardation plate when vectors $S_{IN}^{(1)}$, $S_{IN}^{(2)}$ and $S_{IN}^{(3)}$ are incident. Values (in degrees) for the azimuth and retardation are 14.71, 15.10 and 14.95, and 93.23, 90.43 and 89.98, respectively. These results are close to those presented in table 1.

At this point the remaining question is: what is the error in the determination of the azimuth and retardation using equations (9) and (10) when polarization properties such as depolarization and/or diattenuation are not negligible as expected? To answer this question we have simulated an optical system composed of a birefringent plate ($\beta = 35^\circ$, $\Delta = 80^\circ$) followed by (1) a diattenuator (partial linear polarizer) with different amounts of diattenuation and (2) a depolarizer. Figure 1 shows the error in the calculation of β and Δ for different amounts of diattenuation and depolarization, respectively. Errors in retardation and azimuth when both effects are present are presented in figure 2.

If depolarizing effects are present in the system, the DOP of the emergent beam decreases ($DOP = g < 1$) and

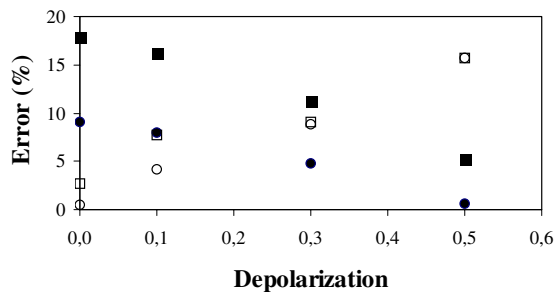


Figure 2. Errors in the determination of the retardation (full symbols) and azimuth (open symbols) of a birefringent sample with the effects of diattenuation and depolarization combined together. Circles and squares correspond to diattenuations of 0.15 and 0.30, respectively.

equations (9) and (10) become respectively

$$\beta = \frac{1}{2}a \tan\left(\frac{g - S_1^{(1)}}{S_2^{(1)}}\right) \quad \Delta = a \cos\left(1 - \frac{2S_2^{(1)}}{g \cdot \sin(4\beta)}\right)$$

$$\beta = \frac{1}{2}a \tan\left(-\frac{g + S_1^{(2)}}{S_2^{(2)}}\right) \quad \Delta = a \cos\left(1 + \frac{2S_2^{(2)}}{g \cdot \sin(4\beta)}\right)$$

$$\beta = \frac{1}{2}a \tan\left(\frac{S_1^{(3)}}{g - S_2^{(3)}}\right) \quad \Delta = a \cos\left(1 - \frac{2S_1^{(3)}}{g \cdot \sin(4\beta)}\right). \quad (11)$$

When using a linear polariscope, just the DOLP can be calculated and only if S_3 is close to zero [27] will $DOP = DOLP = g$.

3. Polarimetry in the human eye using a linear polariscope

Since previous workers reported that the most important polarization property in the human eye is retardation associated with the birefringence, this could be thought of as a pure linear retarder (the slight effects of diattenuation and depolarization being much smaller than that of the birefringence). Using the theoretical schemes of section 2, the results of this assumption can be compared to those obtained using complete polarimetry in order to test the accuracy of this approximation.

3.1. Methods: experimental apparatus and procedure

Information about the polarization properties of the living human eye was extracted from the central part (2.3 min of arc in radius) of double-pass retinal images. The experimental set-up used for this purpose incorporates both a Mueller matrix polarimeter and a linear polariscope into an ophthalmoscopic double-pass apparatus [28]. For the calculation of the full Mueller matrix, the generator and analyser units (placed in the incoming and registration pathways, respectively) consisted of two liquid-crystal variable retarders and two removable quarter-wave plates (MM configuration) (see [11, 12] for more details). For incomplete polarimetry two rotating linear polarizers acted as generator and analyser units (PA configuration). A schematic representation of the experimental system is depicted in figure 3. Briefly, a light beam (633 nm) passes through the generator unit and enters the eye, forming

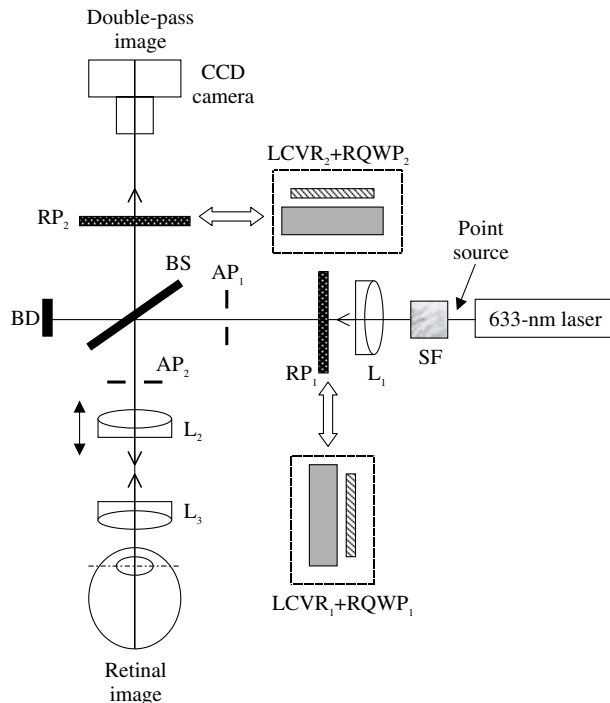


Figure 3. Simplified diagram of the double-pass polarimeter for configurations MM and PA. RP₁ and RP₂, rotating linear polarizers; LCVR₁ and LCVR₂, liquid-crystal variable retarders; RQWP₁ and RQWP₂, removable quarter-wave plates; SF, spatial filter (composed of a pinhole and a microscope objective not shown in the figure); L₁, L₂, and L₃, achromatic doublets; AP₁ and AP₂, artificial pupils; BS, pellicle beam splitter; BD, black diffuser.

the image of a point source. In the exit pathway the back-reflected light goes through the analyser unit and reaches a CCD camera. Artificial pupils (AP₁ and AP₂) act as effective entrance and exit pupils, respectively. An afocal system (lenses L₂ and L₃) and an optional trial lens permit the correction of ocular ametropias.

A series of double-pass retinal images (16 for MM and 9 for PA) of a point source (4 s exposure time, 2 mm pupil diameter) were recorded for independent combinations of polarization generator–analyser settings. Measurements were carried out in two young normal subjects.

From the 16 images recorded with the MM configuration the Mueller matrix was calculated and the parameters of polarization extracted using the polar decomposition theorem of Lu and Chipman [29] as previously reported [11, 12]. When using the PA configuration, for each fixed incoming linear polarization state three images corresponding to three independent orientations of the analyser (horizontal, 45° and vertical) were registered. From each set of three images, retardation and azimuthal angles of the ocular birefringent structure were computed using equations (9) and (10).

3.2. Results

3.2.1. Retardation and azimuth of the living human eye.

If ocular polarization properties, such as diattenuation and depolarization, are not negligible compared to birefringence, the results of considering the human eye just as a retardation plate will be wrong and very different from those obtained from

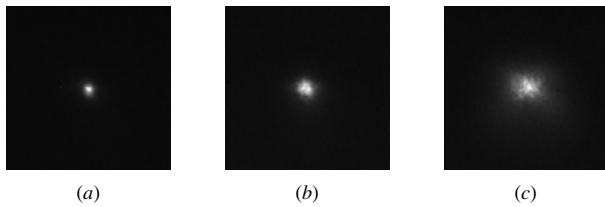


Figure 4. Double-pass retinal images (subject 1, 2 mm pupil diameter) recorded for a fixed incoming linear horizontal polarization state and three different orientations of the analyser: (a) horizontal, (b) 45°, (c) vertical. Each image subtends 59 min of arc and are normalized to each maximum.

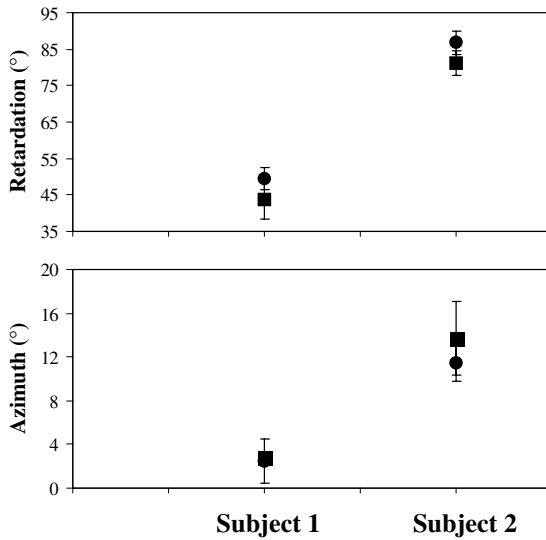


Figure 5. Retardation and azimuth for the central part of 2 mm double-pass images in two observers obtained using the information given by the Mueller matrix (circles) and considering the eye as a pure retarder (squares). Error bars indicate standard deviation. The reference axis is horizontal and angles are positive in the anti-clockwise direction.

the Mueller matrix. To check if the hypothesis of considering the human eye as a retarder is reasonably correct, in the following we compare the results of ocular retardation and azimuth extracted from the Mueller matrix and those obtained with the method described in section 2 (equations (9) and (10)).

As an example, figure 4 shows double-pass retinal images registered with the PA configuration. The images correspond to a fixed incoming polarization state (linear horizontal) and three orientations of the analyser. Double-pass images for the 16 combinations of generator–analyser in the MM configuration can be seen in [30].

Figure 5 presents the results of retardation and azimuth calculated at the central part of the double-pass images in two different subjects using both methods. Error bars overlap for all parameters and subjects but for one (retardation in subject 2). Although we only have two subjects and three measurements in each series, some statistics (t test) were performed in order to better understand the behaviour of the data we obtained. Results show that differences between the parameters calculated using the two methods are not significant ($p > 0.05$). However, $p = 0.036$ for the case cited above.

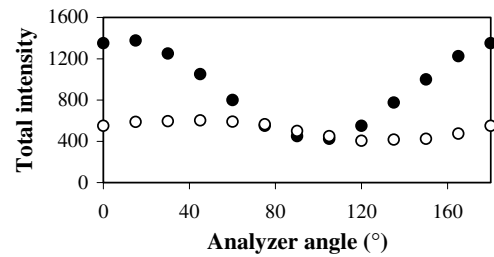


Figure 6. Effect of changing the angle of the analyser on the retinal image intensity for two different image locations: core (black circles) and tails (white circles).

3.2.2. Effect of a rotating linear polarizer used as an analyser.

In an additional experiment, double-pass retinal images were registered with an FPA configuration as follows: a fixed (horizontal) linear polarizer in the illumination channel (first pass) and a second (rotating) linear polarizer (analyser) in the detection channel (second pass). The orientation of the transmission axis of the analyser was systematically rotated over 180° and a double-pass retinal image was registered for each orientation (in increments of 15°). This experimental configuration measures the effect of rotation of an analyser (linear polarizer) on the intensity of the emergent light when incident linear polarized light is used.

It has been suggested [21–23, 31] that the light returning from the retina has basically two components: a directional component (guided through the photoreceptors) and a diffuse component (probably due to the scattering of light not passing through the photoreceptors). In this sense, the effect of the analyser has been studied at two different locations across the double-pass retinal image: core and tails (20 min of arc from the centre of the image). Figure 6 shows the results in subject 1. Each intensity value corresponds to the total intensity in a circle subtending 6 min of arc.

For the core of the images there are large variations in intensity when the angle of the analyser is changed (modulation of 0.53). However, for the edges only minor intensity variations are produced (modulation of approximately 0.20). This indicates that changes in the polarization state of the emergent light are different and depend on the area of the image. Although, between parallel (horizontal–horizontal) and crossed (horizontal–vertical) linear polarizers the intensity clearly reduces for the central part of the image, extinction does not occur.

4. Discussion and conclusions

4.1. Polariscope using linear polarizers

The advantages and disadvantages of using linear polarizers in polarimetry have been described. Only nine elements of the Mueller matrix can be calculated using a linear polariscope. These elements are mainly related to dichroic properties (linear polarization) such as linear diattenuation, the axis of major transmittance and transmission coefficients. These parameters are associated with the intrinsic structure of the optical system and could be very useful in characterization of materials and biological samples, giving information about the preferential absorption of light polarized in a particular

direction (as a consequence of both internal molecular arrangement and distribution of the indices of refraction). Linear diattenuation (or linear polarization sensitivity) is often specified as a performance parameter in remote sensing designed to measure incident power independently of any linearly polarized component present in the scattered earthlight [32]. Missing elements (m_{3i} and m_{i3}) contain partial information on the retardation and azimuth due to birefringence, as well as circular diattenuation.

When a linear birefringent sample presents only small amounts of other forms of polarization, that is, birefringence is the most important polarization property, the polarization parameters associated with this birefringence (retardation and azimuth) can be computed using a much simpler procedure (three measurements of intensity) than the calculation of the complete Mueller matrix. Here, parameters are computed by means of two easy equations as explained in detail in section 2. As a first example we have used these expressions to calculate the azimuth and retardation of a $\lambda/4$ plate (see table 1). Errors were lower than 3%. In general, any non-dichroic linear retarder (e.g. some crystals such as quartz) or some birefringent samples (e.g. some physiological liquids or biological tissues) can also be analysed with this procedure.

However, the accuracy of the method depends on the different amounts of depolarization and diattenuation present in the system. If birefringence is combined with diattenuation or depolarization separately, the error in the determination of the parameters increases when increasing the amount of diattenuation or depolarization, as shown in figure 1. In particular, when depolarization exists, errors in retardation are much larger than those corresponding to azimuth. Figure 2 shows the results when both effects are present in the sample: for a fixed diattenuation, whereas errors in the calculation of the azimuth increase with depolarization, errors corresponding to retardation decrease.

Although the use of a rotating analyser gives information on the polarization properties of light beams and optical systems, this configuration could be misleading when characterizing the polarization state of a light beam (see the appendix). Whereas linear polarizers are cheap simple optical elements, high quality compensators (liquid-crystal variable retarders, Pockels cells, photoelastic modulators) are much more expensive. Moreover, their calibration requires careful and accurate operation.

On the other hand, the eyes of many insects, fishes and birds are basically analysers of polarized light which are used as a compass for navigation and migration [33–35]. Only a few humans can detect different types of polarized light [10]. Tensiscopes (composed of two crossed linear polarizers) are used to test, in a qualitative way, the stress in lenses mounted in spectacles. The use of linear polarizers has also been very useful in techniques for observation and detection through scattering media [36].

4.2. The human eye as a linear retarder

Ocular polarization properties have been previously reported (see references in the introduction). Although the retardation and azimuthal angles of the eye depend on the subject, birefringence (the living eye and *in vitro* cornea, lens and

retina) has been found to be linear [10, 12, 14, 24, 37, 38]. Mathematical models [39, 40] also agree with these results.

Mueller matrix polarimetry in the human eye has shown a substantially DOP preservation [21, 22, 24, 25, 41]. Values of foveal diattenuation between 0.06 and 0.15, depending on the wavelength of the incident light, have been reported [14, 25, 42–44].

In view of this, in the method presented in this paper the human eye has been considered as a simple retardation plate. We have compared the results of retardation and azimuth at the central part of double-pass images obtained using both full Mueller matrices and the approximation presented here (figure 5). The 16 measurements required to calculate the Mueller matrix are reduced to three. This decreases the time needed for measurements and increases the comfort of the observer. Increments (absolute value of the difference between the results for MM and PA configurations) of about 6° and 2° for retardation and azimuth were found. In general, for this kind of subject these differences were not significant. Larger standard deviations (such as those corresponding to retardation in subject 2) might be a result of small differences in the centring of the subject among exposures and non-controlled fluctuation of the accommodation, among others. When using equations (11) that include the experimental calculated DOLP, errors in azimuth remain similar but errors in retardation decrease (values of 45.6° and 84.3° for retardation in subjects 1 and 2, respectively).

At this point we need to take into account that the work presented here corresponds to young healthy eyes. These results may not be typical of all human eyes, due to changes in the optics of the eye with age, surgery or pathologies.

4.3. Effect of a rotating analyser in double-pass retinal images

The effect of rotating the analyser over 180° on the intensity of the reflected light has been shown for double-pass retinal images. As expected, variations in the emergent intensity as a function of the angle of the analyser occur as a result of the ocular polarization properties. However, changes in the central part of the retinal-point image are very different from changes in the edges. In the central part of the image the intensity of the emerging light strongly depends on the angle of the analyser. Despite the complicated polarization properties of the eye, a maximum in the emergent intensity for the central area of the image appears when the transmission axes of the polarizer and analyser are parallel. This means that the polarization ellipse of the emergent light has changed its ellipticity φ ($\varphi = 0$ for the incoming polarization state) after double-passing the eye. However, changes in the azimuth χ seem to be minor. Changes in azimuthal angle are often associated with the presence of noticeable diattenuation. This indicates that this polarization property hardly affects the polarization state of the light going through the ocular media. These results are similar to the effect of a retardation plate placed between two linear polarizers [42] and confirm that the eye basically acts as a linear retarder whose polarization parameters (retardation and azimuth angle) depend on the observer.

The tails of the images correspond to the light emerging from the eye that has suffered scattering processes and intraocular diffusion. In this sense the DOP for that area is

low [41]. In that part of the image there are small changes in the intensity when rotating the analyser and the angle producing the minimum intensity varies. The ratio of maximum to minimum intensity for the core of the images is more than two times larger than for the edges (see figure 6) which indicates the difference in DOP for both zones.

Point-by-point studies (not imaging) by Charman [22] suggested that, for all visible wavelengths, the light maintaining the DOP after retinal reflection could be registered using parallel polarizers but crossed polarizers are required to register depolarized light. As stated previously, this is not totally correct (see, for instance, [41]). However, these results provided information about the different contributions of the exiting light. Linear polarizers were also used to find the origin of the two components (directional and diffuse) of the returning light. However, it has been recently shown that the major retinal layer where the light is reflected does not depend on the polarization state of the incident light [41].

Burns and co-workers [23] presented similar results for the pupil's plane: a (guided) fraction of the light coming back from the ocular fundus fills only a portion of the pupil, whereas the other component (scattered) fills the entire pupil. Measurement of the DOP for the pupil's plane also represents a useful tool to measure directional properties of human photoreceptors [41].

In clinical applications, crossed polarizers have been reported to enhance the visibility of the arcuate bundles of the retinal nerve fibre layer [45]. This configuration has also been used to observe the corneal, lenticular and macular crosses [46, 47]. Lenticular features which are not discernible when using standard illumination can be seen when polarizers are incorporated into a biomicroscope [48]. Some clinical devices also used differential polarization detection based on linear polarization [18].

To summarize, ocular polarimetry which is used mainly to measure retinal thickness requires a generator and an analyser of polarization states composed of a compensator and a linear polarizer. As an easy alternative to these classical polarimeters we propose an imaging linear polariscope (PA configuration) to calculate the polarization properties of the living human eye. The retardation and azimuth for the central part of the cornea-fovea obtained with this PA configuration have been compared to those obtained from the Mueller matrix in the same subjects. The results indicate that, in terms of polarization and for young healthy eyes, considering the human eye as a linear retarder is a fairly good approximation. Whereas light corresponding to the central part of the double-pass retinal point spread function keeps the DOP, the edges of the image are almost depolarized. The detailed analysis of this area will give information about retinal and intraocular scattering. The method presented in this work can be adapted to a clinical environment in order to study changes in the corneal structure after refractive surgery or corneal transplantation. Any nondichroic linear retarder such as birefringent biological samples, media with stress- (load-) induced birefringence or crystals can also be studied.

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Appendix. The use of a rotating polarizer as an analyser

Many optical systems change the polarization state of the incident light in a complicated way. Although relevant information can be obtained by using a PA system, sometimes this is not the most appropriate set-up for the study. In this appendix we will show how such a configuration would erroneously identify completely polarized states as partially depolarized.

Let us consider a Stokes vector, S , associated with a partially polarized light beam emerging from an optical system. This vector can be considered as a superposition of two components [49]:

$$S = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = S_N + S_P = \begin{pmatrix} I_N \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} I_P \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (\text{A.1})$$

where S_P and S_N are Stokes vectors associated with totally polarized (elliptically polarized in general, and linearly or circularly polarized in particular) and depolarized fractions respectively. I_P and I_N satisfy $I_P = (S_1^2 + S_2^2 + S_3^2)^{1/2}$ and $I_N = S_0 - I_P$:

$$\begin{aligned} S_N &= \{(S_0 - [S_1^2 + S_2^2 + S_3^2]^{1/2}), 0, 0, 0\}^T \\ S_P &= \{[S_1^2 + S_2^2 + S_3^2]^{1/2}, S_1, S_2, S_3\}^T. \end{aligned} \quad (\text{A.2})$$

Taking into account the definition of DOP, S can be expressed as

$$\begin{aligned} S &= (1 - \text{DOP}) \cdot \begin{pmatrix} S_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \text{DOP} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \\ &= S_0 \cdot \begin{pmatrix} 1 \\ \text{DOP} \cdot \cos 2\chi \cdot \cos 2\varphi \\ \text{DOP} \cdot \sin 2\chi \cdot \cos 2\varphi \\ \text{DOP} \cdot \sin 2\varphi \end{pmatrix}. \end{aligned} \quad (\text{A.3})$$

If this light beam passes through a rotating linear polarizer (acting as an analyser) before reaching a recording stage, the Stokes vector corresponding to the detected light will be

$$\begin{pmatrix} S_0^{(p)} \\ S_1^{(p)} \\ S_2^{(p)} \\ S_3^{(p)} \end{pmatrix} = \frac{I_N}{2} \cdot \begin{pmatrix} 1 \\ c \\ s \\ 0 \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} I_P + c \cdot S_1 + s \cdot S_2 \\ c \cdot I_P + c^2 \cdot S_1 + c \cdot s \cdot S_2 \\ s \cdot I_P + c \cdot s \cdot S_1 + s^2 \cdot S_2 \\ 0 \end{pmatrix} \quad (\text{A.4})$$

with $c = \cos 2\alpha$, $s = \sin 2\alpha$, α is the azimuth of the transmission axis of the analyser and $S_0^{(p)}$ is the registered intensity:

$$S_0^{(p)} = I_\alpha = \frac{1}{2} \cdot (I_N + I_P + c \cdot S_1 + s \cdot S_2). \quad (\text{A.5})$$

In particular, if the transmission axis of the analyser is horizontal ($\alpha = 0^\circ$), vertical ($\alpha = 90^\circ$) and at 45° , the

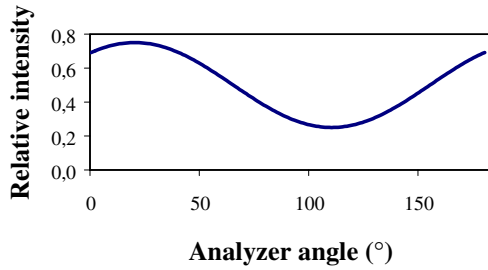


Figure 7. Pattern of intensity corresponding to an elliptical totally and partially polarized light beam when using a rotating linear polarizer as an analyser.

intensities will be

$$\begin{aligned} I_h &= \frac{1}{2} \cdot (I_N + I_p + S_1) = \frac{1}{2} \cdot (S_0 + S_1) \\ I_v &= \frac{1}{2} \cdot (I_N + I_p - S_1) = \frac{1}{2} \cdot (S_0 - S_1) \\ I_{45} &= \frac{1}{2} \cdot (I_N + I_p + S_2) = \frac{1}{2} \cdot (S_0 + S_2). \end{aligned} \quad (\text{A.6})$$

These equations show that both the totally polarized and depolarized portions contribute to the final intensity detected by the recording stage. Moreover, if light corresponding to the vector S was totally polarized ($I_p = S_0^{(p)}$), equation (A.5) would be exactly the same. This confirms that a rotating linear polarizer used as an analyser cannot separate the two contributions and partially polarized light (non-null I_N) would be interpreted as elliptical totally polarized light or vice versa.

In the following a numerical example is presented in order to clarify this fact. Let us suppose that a totally polarized light beam ($\varphi = 30^\circ$ and $\chi = 20^\circ$) emerges from a system under study ($I = 1$). In this case $S_{TP} = [1, 0.383, 0.321, 0.866]^T$ and the registered intensity pattern as a function of the axis of the analyser is shown in figure 7. However, if the beam is partially polarized (i.e. $\text{DOP} = 0.75$) with the vector $S_{PP} = [1, 0.287, 0.241, 0.650]^T$, the totally polarized fraction having the same φ and χ , the intensity signal is the same as in figure 7. S_3 cannot be determined in either case and, whereas $\text{DOLP} = 0.5$ for S_{TP} , the parameter is 0.38 for S_{PP} . The DLOP does not indicate if the analysed light is totally or partially polarized.

A.1. Polarization parameters of a light beam by using a rotating polarizer as analyser

With the three measurements of intensity shown in equation (A.6) only a partial reconstruction of the Stokes vector (three elements) is achieved:

$$\begin{aligned} S_0 &= I_h + I_v \\ S_1 &= I_h - I_v \\ S_2 &= 2I_{45} - I_h - I_v. \end{aligned} \quad (\text{A.7})$$

Elements in equation (A.7) allow the calculation of the degree of linear polarization (DOLP) of the beam and the azimuth of the vector (χ):

$$\text{DOLP} = \frac{(S_1^2 + S_2^2)^{1/2}}{S_0} \quad \chi = \frac{1}{2} a \tan\left(\frac{S_2}{S_1}\right). \quad (\text{A.8})$$

Neither φ nor DOP can be computed when the beam is, in general, partially polarized. For the particular case of a totally polarized beam the value of 2φ can be calculated as

$$2\varphi = a \cos\left(\frac{S_1}{S_0 \cdot \cos(2\chi)}\right). \quad (\text{A.9})$$

However, because in the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ where the ellipticity of the polarization ellipse is defined, the cosine function satisfies $\cos(2\varphi) = \cos(-2\varphi)$, there will be an indeterminacy in the sign of the ellipticity (positive and negative for the upper and lower hemispheres of the Poincaré sphere) and so in S_3 :

$$S_3 = \pm S_0 \cdot \sin\left(a \cos\left(\frac{S_1}{S_0 \cdot \cos(2\chi)}\right)\right). \quad (\text{A.10})$$

This implies that, even with totally polarized light, the fourth component of the Stokes vector cannot be computed when using a PA system. For natural systems this is not very important, because S_3 is very small [31]. However, for other optical systems this could not happen.

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