Statistical description of wave-front aberration in the human eye

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Received August 8, 2001

The wave aberration of the human eye has been measured by means of a Hartmann–Shack wave-front sensor in a population of normal subjects. The set of data has been used to compute the phase distribution, the power spectrum, and the structure function for the average eye to analyze the statistics of the ocular aberration considered as a phase screen. The observed statistics fits the classical Kolmogorov model of a statistically homogeneous medium. These results can be of use in understanding the average effect of aberrations on the retinal image and can serve as a tool to analyze the consequences of ocular-aberration compensation by adaptive optics, customized ophtalmic elements, or refractive surgery.

OCIS codes: 330.7310, 170.4460.

For more than two centuries, researches have been interested in measuring the degradation of the retinal image in the human eye beyond refractive errors. The wave aberration (WA) function is the most convenient way of describing the performance of ocular optics.[1] The use of compact and reliable wave-front sensors[2–4] to determine the eye's aberrations opens a new way to understand the behavior of the eye as a phase screen. It allows the measurements of a large population for statistical analysis of the ocular optics. In this Letter, we study the statistical properties of the eye's optics, which are assumed to be a phase screen. This approach differs from those of previous statistical studies that were based on the evaluation of individual modes.[5] We show that the eye behaves as a statistically homogeneous medium, which follows a Kolmogorov model of the power spectrum. Consequently, light propagation through the eye resembles propagation through other living tissues[6] or through the atmosphere.[7] One can explain this effect by considering the eye as being composed of structures with dimensions ranging from possibly 100 nm to millimeters (we denote these structures as the inner, l_0, and the outer, L_0, scales of the eye, respectively). This statistical description of the eye can be useful for such tasks as estimating the averaged retinal point-spread function, performing numerical simulations of ocular-aberration compensation, studying different strategies for refractive surgery, and the development of contact or intraocular lenses.

A Hartmann–Shack (H–S) sensor was used to measure the ocular WA. The apparatus is based on another described in Ref. 4. A beam splitter is used to combine an illumination channel, basically consisting of a spatially filtered low-coherence laser diode conjugate to the subject's retina, and a measurement channel, where the light emerging from the retinal reflection is directed to a H–S sensor. This sensor is composed of a microlens array that is conjugate to the subject's pupil, with a CCD detector on its focal plane. As a result, an image consisting of a series of spots can be recorded. The displacement of each spot with respect to its reference position is proportional to the local slope of the wave front, and one can easily use the displacement to obtain a modal expansion of the WA. In our case, we use the Zernike circle polynomials as the functional basis for this expansion. The apparatus is completed with a system that controls the pupil position and a fixation channel.

The main differences with respect to previous apparatuses are the use of a low-coherence laser diode for illumination and a short-focal-length lenslet array for the H–S sensor. The first feature allows recording of short-exposure frames without the speckle noise that is present when a coherent source is used. The second produces an increased dynamic range for our sensor and a reduction of the retinal irradiance.

Theoretically, the WA should be expressed as a weighted sum of Zernike polynomials[8] with infinite terms:

$$\phi(\mathbf{r}) = \sum_{i=1}^{\infty} a_i Z_i(\mathbf{r}), \tag{1}$$

where a_i are the coefficients of the corresponding Zernike polynomials. In practice, only a finite number of terms are considered. On the one hand, the expansion is truncated to a certain index k. Ultimately, this truncation is imposed by the limited number of sampling points on the pupil plane, but it is well justified by the observed decrease in the relative contribution of increasing-order components to the ocular aberration, as was reported in Ref. 3. On the other hand, our wave-front sensor is unable to measure piston and tilt terms (Zernike polynomials 1–3). Furthermore, when one is analyzing the ocular aberrations, it is usual to investigate the effects of

0146-9592/02/010037-03$15.00/0

high-order terms alone, which is done by cancellation of the low-order Zernike coefficients. This is equivalent to considering the system to be composed of the eye and an ideal partial-correction system that perfectly compensates for the first j Zernike coefficients. Since the Zernike polynomials are orthonormal over a unit pupil, the residual wave-front distortion, defined as the average variance of the wave-front surface, can be calculated as a sum of squared Zernike coefficients. Zernike coefficients up to eighth order ($k = 45$) were measured in 84 healthy eyes over a 7-mm pupil. Measurements were carried out under natural accommodation and pupil conditions, with the fixation target at infinity. The subjects' ages ranged from 20 to 29 years. Retinal irradiance was always several orders of magnitude below ANSI standards. From the WA maps, the histogram of phase values across pupil positions and subjects was calculated. Once it is normalized, this histogram is an estimate of the phase probability density and was found to follow a zero-mean Gaussian distribution. Figure 1 shows these results for two compensation levels: $j = 3$, which means that piston, tip, and tilt are compensated for, and $j = 6$, which means that the first three modes, along with defocus and astigmatism, are compensated for. Since the mean value of the distribution is zero, it is fully described by its second moment. Consequently, the power spectrum and the structure function, defined as $\langle[\phi(\mathbf{r}) - \phi(\mathbf{r}')]^2\rangle$,[9] suffice to characterize light propagation through the eye.

The experimental power spectra for two different values of j are represented in Fig. 2. Their shape resembles the Von Karman spectrum.[7] This kind of spectrum corresponds to a statistically homogeneous medium following the Kolmogorov theory, with phase fluctuations produced by index inhomogeneities that continuously range in size between two finite scales, l_0 (inner scale) and L_0 (outer scale). For frequencies below the reciprocal of the outer scale, the Von Karman spectrum is flat. Between this value and the reciprocal of the inner scale, it follows a $-11/3$ power law. For frequencies above $1/l_0$, the Von Karman spectrum vanishes.

The experimental power spectra in Fig. 2 are represented for only a limited range of spatial frequencies. This range is imposed by the Zernike terms used for expressing the WA in each case. Each power spectrum presents a power-law zone that can be satisfactorily fitted with a $-11/3$ exponent even though noise is present. Furthermore, the low correction spectrum remains flat for frequencies below 0.2 mm^{-1}. According to the Kolmogorov theory, the outer scale of the index inhomogeneities can be calculated as the reciprocal of this value, i.e., 5 mm. This result cannot be checked with the high-order correction power spectrum, since it is not well defined below 0.3 mm^{-1}. Similarly, the frequency range in which the low- and high-order correction power spectra are defined is too short in either case to allow determination of the inner inhomogeneity scale. To increase this range, one should use a higher number of coefficients, meaning that a sensor with a higher number of subapertures would be required. However, it does not seem feasible to reach l_0, even with a system that stretches current technology to its limits.

It is important to determine whether the eye is a statistically homogeneous medium. For this purpose, we analyzed three parameters that characterize the eye as a phase screen: the residual phase variance, the coherence area, and the correlation length. The first parameter can be directly obtained from the measurements of the WA function. The other two are estimated from the structure function.

The radial evolution of the phase variance for two different degrees of compensation is shown in Fig. 3. From the behavior of this variance in the low-compensation case, the phase could be thought to be a statistically inhomogeneous process. However, it has been shown that low-order Zernike compensation induces this kind of behavior in a statistically homogeneous medium, since it limits the range of phase fluctuation in the center of the pupil more severely.[10,11] Further compensation quickly leads to a recovery of homogeneity (as a rule of thumb, third-order compensation is enough). This result is in agreement with the results in Fig. 3, in which the curve corresponding to 15 corrected modes shows that the phase variance is no longer a function of position in the pupil plane.

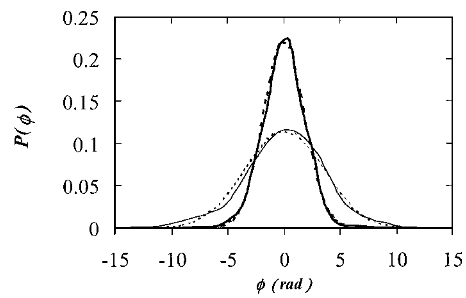

Fig. 1. Distribution of the phase across subjects and pupil positions for a 7-mm pupil diameter, for two different levels of compensation (thin curves, $j = 3$; thick curves, $j = 6$). Solid curves, normalized experimental density functions; dashed curves, Gaussian fits.

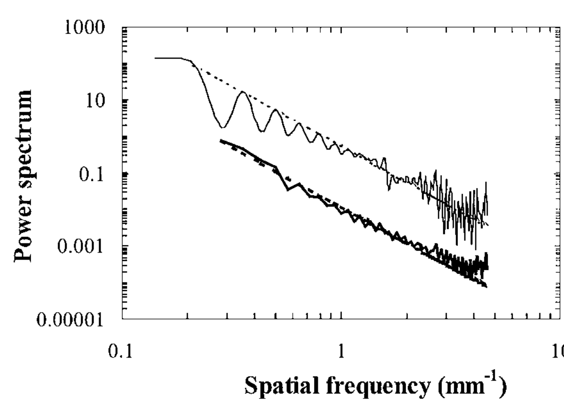

Fig. 2. Power spectra for two different compensation levels (thin solid curves, three modes corrected, thick solid curves, ten modes corrected), compared with a $-11/3$ power law (thin dotted curves, three modes corrected; thick dashed curve, ten modes corrected).

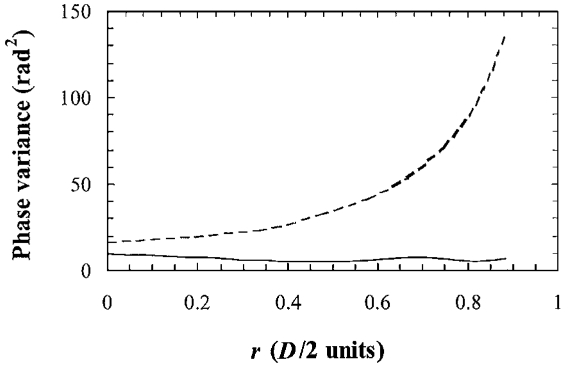

Fig. 3. Radial evolution of the phase variance for two compensation levels (solid curve, three modes corrected; dashed curve, ten modes corrected).

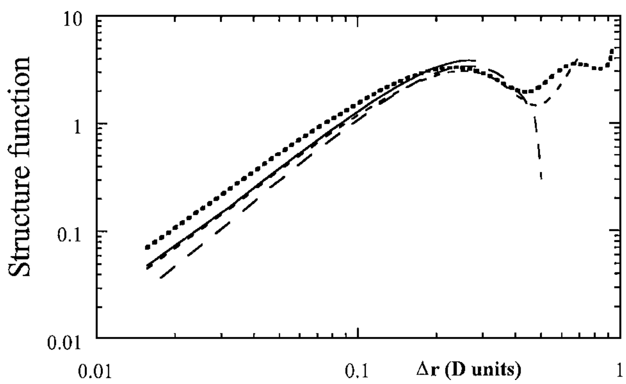

Fig. 4. Structure function calculated with concentric circles of different diameter (d). The pupil diameter is fixed ($D = 7$ mm), and the distance between points (Δr) is in D units. Long-dashed curve, $d = 0.9D$; solid curve, $d = 0.7D$; circles, $d = 0.5D$; short-dashed curve, $d = 0.3D$.

To check this point further, we evaluated the structure function corresponding to the case of 15 corrected modes inside centered circular regions with radii smaller than the pupil. Results for four increasing radius zones are shown in Fig. 4. In all cases, the structure function curve was found to be composed of two parts: a power-law zone that saturates into a noisy segment around a constant value. The saturation value was always approximately equal to twice the average phase variance, $2\Delta_j$. The correlation length, l_c, can be defined as the distance that produces this transition in the structure function behavior.[11] In Fig. 4, it is shown that, for a fixed compensation level, l_c does not change radially. The slope of the power-law zone can be related to a parameter, ρ_0, which can be interpreted as the radius of the coherence area in the pupil (analogous to the Fried parameter in atmospheric phase screen analysis[11]). For a statistically homogeneous medium, this slope is independent on the pupil size. This can be the case with the ocular WA after 15-mode compensation, since the slope changes observed in Fig. 4 are small and can probably be explained in terms of measurement noise.

To conclude, we have studied the statistics of the eye as a phase screen from the experimental measurement of the ocular WAs in a large population. Across subjects and pupil locations, the phase-value behavior was found to closely resemble a Gaussian distribution. The phase power spectrum, the radial evolution of the phase variance, and the structure function were calculated. Analysis of these functions strongly suggests that the eye can be considered a statistically homogeneous medium following Kolmogorov theory with finite outer and inner scales. A value of 5 mm was found for the outer scale, whereas determination of the inner scale is beyond the capabilities of our WA measurement system. The analysis presented in this Letter constitutes a first step toward the statistical characterization of the image formation process in the eye and how it is affected by partial aberration compensation. Furthermore, analysis of the structure function, which will make possible a deeper knowledge of the eye's WA statistical behavior, could be of use for designing ophthalmic or adaptive-optics-based elements for high-order aberration correction, or in the improvement of refractive surgery strategies.

This research was supported by Dirección General de Enseñanza Superior grant AYA2000-1565-C02 to M. P. Cagigal and grant PB97-1056 to P. Artal. M. P. Cagigal's e-mail address is perezcm@correo.unican.es.

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