Institute of Physics Publishing — Journal of Optics A: Pure and Applied Optics

J. Opt. A: Pure Appl. Opt. **3** (2001) 470–476 PII: S1464-4258(01)27581-3

# Indices of linear polarization for an optical system

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Received 7 August 2001, in final form 17 September 2001
Published 9 October 2001
Online at stacks.iop.org/JOptA/3/470

**Abstract**
Direct ($G_L$) and reverse ($G_{LR}$) indices of linear polarization for an optical system are presented. These parameters have been calculated using the concept of the degree of linear polarization for the light beam emerging from the system and they are expressed as a function of the elements of the corresponding Mueller matrix. Values of $G_L$ and $G_{LR}$ for pure and combinations of polarization elements have been calculated. Real examples such as the human eye and an *in vitro* cornea are also shown.

**Keywords:** Degree of polarization, linear polarization, Mueller matrix

## 1. Introduction

Polarization is an intrinsic property of light. It makes reference to the curves described by the electric field vector on the plane perpendicular to the direction of the propagation of the light beam [1, 2]. These curves correspond to the different polarization states of the light (linear, circular and elliptical). When the electric field vector vibrates in all directions with no preferential orientation, the light is depolarized. Light containing both polarized and depolarized components is known as partially polarized. Whereas Stokes vectors can describe any polarization state, Jones vectors are only useful for totally polarized beams [1]. The degree of polarization (DOP) [3] is the parameter related to that property of the light. For a light beam with a Stokes vector $S = (S_0, S_1, S_2, S_3)^T$ is defined as the ratio of the polarized-component intensity to the total intensity [3]:

$$\mathrm{DOP} = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0} \tag{1}$$

where $S_0$ is the total intensity of the beam, and $S_1$, $S_2$ and $S_3$ are the differences in intensity between linear horizontal and linear vertical, linear at +45° and linear at −45°, and right and left circularly polarized components, respectively. DOP ranges from zero (depolarized light) to unity (totally polarized light). If $0 < \mathrm{DOP} < 1$ the beam is partially polarized. The degree of linear polarization (DOLP) of a light beam is defined as [3]:

$$\mathrm{DOLP} = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}. \tag{2}$$

These parameters are very useful tools in different fields such as meteorology, astronomy, ophthalmology and research in optical fibres among others [4–7].

Equations (1) and (2) refer to the intrinsic polarization (depolarization) properties of a light beam, however, sometimes it is more useful to know about the depolarization effects of an optical system itself. In this sense, depolarization is understood as a process which couples polarized light into depolarized light. Both concepts are strongly linked and the Stokes–Mueller formalism [1] is required to describe those media. The DOP of a system represented by the Mueller matrix $M = m_{ij}$ $(i, j = 0, 1, 2, 3)$ is given by [3, 8]

$$G_T = \frac{\sqrt{\left(\sum_{i,j=0}^{3} m_{ij}^2\right) - m_{00}^2}}{\sqrt{3}\cdot m_{00}} \qquad (0 \leqslant G_T \leqslant 1). \tag{3}$$

If $G_T = 1$, light emerging from the system will be totally polarized. When $G_T < 1$, the light will be partially polarized. $G_T$ was called the depolarization index by Gil and Bernabeu [8]. Chipman [3] defined the depolarization of the matrix, Dep(M), as $1 - G_T$. Evidently, if Dep(M) $= 0$, the sample does not depolarize the totally polarized incident light.

Experimental systems used to determine polarization properties of light beams and samples are called polarimeters. For a complete determination of the Mueller matrix (or alternatively the Stokes vector) four independent polarization states in both generator and analyser units are required [9]. In order to obtain these states, a fixed linear polarizer and a compensator (quarter-wave plate, variable retarder) are used [3, 9–13]. A polarimeter using only rotating linear polarizers can neither generate circular polarized light nor determine the circular polarization content of a beam. That technique is called incomplete polarimetry [3]. Moreover, when one polarization property is much more important than the rest, the calculation of the 16 elements of the Mueller matrix is not required [14–16].

Although most of the light around us (the Sun, bulbs, ...) is depolarized, linearly polarized light also plays an important role. Light specularly reflected from dielectric surfaces, such as snow, a swimming pool or the ocean, is usually (partial) linearly polarized [17]. Polaroid sunglasses are based on this effect. This fact, combined with the scattering, is what makes blue sky polarized although light coming from the Sun is totally depolarized [18]. Many invertebrates [19, 20] and vertebrates [21–24] detect linearly polarized light which is used as a compass in both navigation and communication dances.

In particular, although the ocular media and the retina have complicated polarization properties, only some humans can detect polarized light (see [25] as a general review). The cornea is highly birefringent, presenting both intrinsic and form birefringence. The lens is slightly birefringent and its contribution to the total ocular effect may be neglected. The retinal structure is more complex and has properties of birefringence, dichroism and depolarization. All these properties should be taken into account, particularly in such applications as fundus reflectometry, measurements of the retinal nerve fibre layer thickness or the estimates of the retinal image quality by using double-pass techniques [26–28]. Although imaging polarimetry [29] allows a more complete description of spatial changes in the polarization state of the light passing the eye, those polarization properties have been studied many times using only linear polarizers [30–32].

If, after passing an optical system under study (i.e. the human eye) with different polarization properties, including partial depolarization, and prior to reaching a recording stage (CCD camera, photomultiplier, ...), the beam passes some polarizing optical elements (most beamsplitters placed in experimental systems are dichroic, lenses sometimes have noticeable birefringent effects) the detected intensity will depend on its polarization state. For instance, let us suppose a total linear polarizer (analyser) placed in front of the recording unit. If the major axis of the ellipse of polarization associated with the incident light (emerging from the system) is not parallel to the transmission axis of the linear polarizer, the detected intensity will be lower than if it is parallel to it [28]. Moreover, when using a rotating polarizer as an analyser, elliptical totally polarized light can be identified as partially polarized light. With that configuration, circular and depolarized light cannot be distinguished either. That is, depending on the polarization (depolarization) effects of a system, the registered signal when an analyser is placed in front of the recording stage could lead to erroneous results. Due to that, sometimes it is interesting to know which fraction of the (in general) elliptical partially polarized emergent light would correspond to linear polarization, as a function of the polarization properties of the system and, more concretely, as a function of the elements of the corresponding Mueller matrix. In this paper we present two parameters, termed the indices of linear polarization for a system ($G_L$ and $G_{LR}$), to identify that portion of the emergent light.

## 2. Index of linear polarization: definition

Let $M_D = m_{ij}$ $(i, j = 0, 1, 2, 3)$ be the Mueller matrix of a general optical system. This matrix will be used to calculate the transformation of six equidistant polarization states on the Poincaré sphere corresponding to totally polarized light: four linear (horizontal, vertical and $\pm 45°$) and two circular (right and left) given by

$$S_+^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad S_+^{(2)} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad S_+^{(3)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$S_-^{(1)} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \qquad S_-^{(2)} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad S_-^{(3)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}. \tag{4}$$

The transformed Stokes vectors are result of

$$S_+^{\prime(i)} = M_D \cdot S_+^{(i)} \qquad S_-^{\prime(i)} = M_D \cdot S_-^{(i)}. \tag{5}$$

If for each $S_{(\pm)}^{\prime(i)}$, $K_{(\pm)}^{(i)} \equiv S_{1(\pm)}^{\prime(i)^2} + S_{2(\pm)}^{\prime(i)^2}$ is defined, we obtain

$$K_+^{(i)} = m_{10}^2 + m_{1i}^2 + 2 \cdot m_{10}m_{1i} + m_{20}^2 + m_{2i}^2 + 2 \cdot m_{20}m_{2i}$$
$$K_-^{(i)} = m_{10}^2 + m_{1i}^2 - 2 \cdot m_{10}m_{1i} + m_{20}^2 + m_{2i}^2 - 2 \cdot m_{20}m_{2i}. \tag{6}$$

The intensity for every emergent Stokes vector $S_{(\pm)}^{\prime(i)}$ will be

$$S_{0(\pm)}^{\prime(i)} = m_{00} \pm m_{0i}. \tag{7}$$

$K'$ and $K'_0$ are defined by the average of the above expressions [8]:

$$K' = \tfrac{1}{6}\sum_{i=1}^{3}(K_+^{(i)} + K_-^{(i)}) = m_{10}^2 + m_{20}^2 + \tfrac{1}{3}\sum_{i=1}^{3} m_{1i}^2 + \tfrac{1}{3}\sum_{i=1}^{3} m_{2i}^2 \tag{8}$$

and

$$K'_0 = \tfrac{1}{6}\sum_{i=1}^{3}(S_{0(+)}^{\prime(i)} + S_{0(-)}^{\prime(i)}) = m_{00}. \tag{9}$$

The parameter $K'$ is the mean of the sum of the squares of the elements corresponding to linear polarization of the emergent light beams. $K'_0$ represents the corresponding averaged intensity.

Using the definition of DOLP in equation (2), an auxiliary index of linear polarization for the system ($G_L^{(\mathrm{aux})}$) is defined as

$$G_L^{(\mathrm{aux})} = \frac{\sqrt{K'}}{K'_0} = \frac{\left(m_{10}^2 + m_{20}^2 + \frac{1}{3}\sum_{i,j=1}^{2}(m_{1i}^2 + m_{2i}^2)\right)^{1/2}}{m_{00}}. \tag{10}$$

For the case of a linear polarizer $G_L^{(\mathrm{aux})} = 2\sqrt{3}/3$. Since the light emerging from a linear polarizer is always linearly polarized, the parameter for that element must be maximum. In view of that we normalize $G_L^{(\mathrm{aux})}$ to have a range between 0 and 1, and the direct index of linear polarization for the system, $G_L$, will be finally defined as

$$G_L = \frac{\sqrt{3}}{2m_{00}}\left(m_{10}^2 + m_{20}^2 + \frac{1}{3}\sum_{i,j=1}^{3}(m_{1i}^2 + m_{2i}^2)\right)^{1/2}. \quad (11)$$

The nine elements involved in the definition of this index of linear polarization are associated with linear diattenuation ($m_{10}$ and $m_{20}$), linear retardance ($m_{12}$ , $m_{13}$ and $m_{21}$), circular retardance ($m_{23}$) and depolarization ($m_{11}$ and $m_{22}$).

Since the polarizing properties of a system can be very different if the direction of the incident and emergent light is exchanged, in the following we calculate the index of linear polarization for the reverse direction. The Mueller matrix $M_R$ describing the system verifies [33]:

$$M_R = Q \cdot M_D^T \cdot Q \quad (12)$$

where

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and $M_D^T$ is the transpose of $M_D$.

Let us suppose that the direction of the light is reversed. Now, the Stokes vector of the outgoing light beam will be

$$S_+^{\prime\prime(i)} = M_R \cdot S_+^{(i)} \qquad S_-^{\prime\prime(i)} = M_R \cdot S_-^{(i)}. \quad (13)$$

Operating in the same way as before:

$$\overline{K}_+^{(i)} = m_{01}^2 + m_{i1}^2 + 2 \cdot m_{01}m_{i1} + m_{02}^2 + m_{i2}^2 + 2 \cdot m_{02}m_{i2}$$
$$\overline{K}_-^{(i)} = m_{01}^2 + m_{i1}^2 - 2 \cdot m_{01}m_{i1} + m_{02}^2 + m_{i2}^2 - 2 \cdot m_{02}m_{i2}. \quad (14)$$

Then

$$K'' = \tfrac{1}{6}\sum_{i=1}^{3}(\overline{K}_+^{(i)} + \overline{K}_-^{(i)}) = m_{10}^2 + m_{20}^2 + \tfrac{1}{3}\sum_{i=1}^{3} m_{i1}^2 + \tfrac{1}{3}\sum_{i=1}^{3} m_{i2}^2 \quad (15)$$

and

$$K_0'' = \tfrac{1}{6}\sum_{i=1}^{3}(S_{0+}^{\prime\prime(i)^2} + S_{0-}^{\prime\prime(i)^2}) = m_{00}. \quad (16)$$

Finally, the index of linear polarization for the reverse direction, $G_{LR}$, will be

$$G_{LR} = \frac{\sqrt{3}}{2m_{00}}\left(m_{01}^2 + m_{02}^2 + \frac{1}{3}\sum_{i,j=1}^{3}(m_{i1}^2 + m_{i2}^2)\right)^{1/2}. \quad (17)$$

Only when $m_{ij}^2 = m_{ji}^2$ (for instance, Mueller matrices corresponding to pure polarization elements such as polarizers or retarders) does $G_L = G_{LR}$.

## 3. Indices of linear polarization for different media

In this section we calculate the direct and reverse indices of linear polarization for different optical systems using the expressions calculated in the previous section. Examples 1–6 refer to ideal polarization elements (alone and in combination), examples 7 and 8 correspond to double-pass experimental measurements of the human eye in [34] and an *in vitro* cornea, respectively. When a system is the result of the combination of different polarization elements, the corresponding Mueller matrix usually does not verify that $m_{ij}^2 = m_{ji}^2$ and the index of linear polarization will depend on the direction of incidence of the light, as will be shown in some of the following examples.

**Table 1.** Indices of linear polarization for different polarization elements: $\theta$, orientation for the transmission axis of the polarizer; $q$ and $r$, intensity transmittances; $\alpha$ and $\delta$, azimuth of the fast axis and retardation for the retarder; $a$, absorption factor; $d$, polarization factor ($(1-d)$, depolarization factor); $\gamma$, angle of rotation.

| Polarization elements | $G_L = G_{LR}$ |
|---|---|
| Linear polarizer $M_p^\theta$ | 1 |
| Linear diattenuator $M_p^0(q, r)$ | $\sqrt{q^2 + r^2}/q + r$ |
| Linear retarder $M_\delta^\alpha$ | $\sqrt{2}/2$ |
| Isotropic absorber $M_a$ | $\sqrt{2}/2$ |
| Partial depolarizer $M_d$ | $d \cdot \sqrt{2}/2$ |
| Rotor $M_R(\gamma)$ | $\sqrt{2}/2$ |

*Example 1. Pure polarization elements.* Taking into account the Mueller matrices associated with different (single) polarization elements [3,35], table 1 presents the values for the corresponding indices of linear polarization.

*Example 2. Combination of a horizontal polarizer and a retarder at 45°.* If a horizontal linear polarizer is followed by a quarter-wave plate with its fast axis at 45° with respect to the horizontal and the light first passes the polarizer, the corresponding Mueller matrix is $M_D^{(1)}$ and $G_L = 0$ (the emergent light will be circularly polarized); however, if the quarter-wave plate is the first element, the matrix will be $M_R^{(1)}$ and $G_L = 1 (= G_{LR}(M_D^{(1)}))$:

$$M_D^{(1)} = M_{\lambda/4}^{45} \cdot M_p^0 = \tfrac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

$$M_R^{(1)} = M_p^0 \cdot M_{\lambda/4}^{45} = \tfrac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

*Example 3. Combination of a depolarizer and a horizontal polarizer.* Let us suppose that an integrating sphere acts as a complete depolarizer with a linear polarizer (transmission axis at angle $\theta$) over one of its ports. When the light is incident on the sphere first, the matrix is $M_D^{(2)}$ and $G_L = \sqrt{3}/2$. Propagating in the opposite direction ($M_R^{(2)}$)$G_L = 0$.

$$M_D^{(2)} = \tfrac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ \cos 2\theta & 0 & 0 & 0 \\ \sin 2\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R^{(2)} = \tfrac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = M_D^{(2)^T}.$$

In a more general way, if the depolarizer is partial (with polarization factor, $d$) the indices of linear polarization will be a function of $d$:

$$G_L(M_D^{(2)}) = \frac{\sqrt{3}}{2}\left(1+\frac{d^2}{3}\right)^{1/2}$$

$$G_L(M_D^{(2)}) = d = G_{LR}(M_D^{(2)}).$$

*Example 4. Combination of a partial depolarizer and a horizontal diattenuator.* If instead of a total polarizer, we have a partial depolarizer followed by a horizontal diattenuator (with intensity transmittances $q$ and $r$) or a horizontal diattenuator followed by a partial depolarizer, the Mueller matrices are respectively:

$$M_D^{(3)} = M_p^0(q,r)\cdot M_d = \frac{1}{2}\begin{pmatrix} q+r & d(q-r) & 0 & 0 \\ q-r & d(q+r) & 0 & 0 \\ 0 & 0 & 2d\sqrt{qr} & 0 \\ 0 & 0 & 0 & 2d\sqrt{qr}\end{pmatrix}$$

$$M_R^{(3)} = M_d\cdot M_p^0(q,r) = \frac{1}{2}\begin{pmatrix} q+r & q-r & 0 & 0 \\ d(q-r) & d(q+r) & 0 & 0 \\ 0 & 0 & 2d\sqrt{qr} & 0 \\ 0 & 0 & 0 & 2d\sqrt{qr}\end{pmatrix} = M_D^{(3)^T}$$

and

$$G_L(M_D^{(3)}) = \frac{\sqrt{3}}{2(q+r)}\left((q-r)^2+\frac{d^2}{3}(q^2+r^2+6qr)\right)^{1/2}$$

$$G_L(M_R^{(3)}) = \frac{d\sqrt{3}}{2(q+r)}\left((q-r)^2+\frac{1}{3}(q^2+r^2+6qr)\right)^{1/2} = G_{LR}(M_D^{(3)}).$$

These expressions are more general than those calculated in example 3, where $q = 1$, $r = 0$ and $d = 0$.

*Example 5. Combination of a linear retarder and a partial depolarizer.* When combining a linear retarder (retardation $\delta$, and azimuth of the fast axis, $\alpha$) and a partial depolarizer, the corresponding Mueller matrix is

$$M_4^{(D)} = M_\delta^\alpha\cdot M_d = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & d(c^2+s^2k) & dsc(1-k) & -dsx \\ 0 & dsc(1-k) & d(s^2+c^2k) & dcx \\ 0 & dsx & -dcx & dk\end{pmatrix}$$

with $c = \cos\alpha$, $s = \sin\alpha$, $k = \cos\delta$ and $x = \sin\delta$. For this case $G_L = G_{LR} = d\cdot\sqrt{2}/2$.

*Example 6. Combination of polarization elements in parallel.* Let us consider the Mueller matrix for an aperture half filled with a vertical polarizer and half filled with a polarizer oriented at 45° ($M_D^{(5)}$). In this case $G_L = G_{LR} = \sqrt{2}/2$.

$$M_D^{(5)} = \frac{1}{2}\begin{pmatrix} 2 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0\end{pmatrix} = M_R^{(5)}.$$

If the aperture is filled with a horizontal polarizer and a quarter-wave plate at 45° the corresponding matrix is

$$M_D^{(6)} = \frac{1}{2}\begin{pmatrix} 3 & 1 & 0 & 0 \\ 1 & 1 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0\end{pmatrix} = M_R^{(6)}$$

and $G_L = G_{LR} = \sqrt{6}/6$.

If the combination in parallel corresponds to a linear polarizer and a partial depolarizer, the corresponding matrix and index of linear polarization are respectively:

$$M_D^{(7)} = \frac{1}{2}\begin{pmatrix} 1 & c & s & 0 \\ c & c^2+d & cs & 0 \\ s & cs & s^2+d & 0 \\ 0 & 0 & 0 & d\end{pmatrix} = M_R^{(7)}$$

$$G_L(M_D^{(7)}) = \frac{\sqrt{3}}{2}\left(1+\frac{1}{3}\left(2d^2+2d+1\right)\right)^{1/2} = G_{LR}(M_D^{(7)}) = G_L(M_R^{(7)}).$$

*Example 7. The human eye.* Using a double-pass imaging polarimeter [29], the spatially resolved Mueller matrix of the human eye in double-pass was obtained. The elements of this matrix were computed from 16 double-pass retinal images (the image of a point source on the retina recorded by a CCD) as broadly explained in [29]. Light forming these images has passed the ocular media twice (cornea, lens and retina). Once the Mueller matrix is known, the parameters of polarization can be extracted. In particular, figure 1(*a*) presents the distribution for the degree of (total) polarization (in double-pass), $G_T$, for a living human eye and 2 mm of pupil. Figures 1(*b*) and (*c*) correspond to the direct and reverse indices of linear polarization. These parameters have been computed from the Mueller matrices (pixel by pixel) using equations (3), (11) and (17). The averaged values for the whole image (subtending about half a degree of the visual field) are $0.45 \pm 0.06$, $0.40 \pm 0.04$ and $0.37 \pm 0.05$ (mean $\pm$ standard deviation) respectively. For the central area (about 7 min of arc) the averaged $G_T$ was $0.78\pm0.07$ and it decreased to $0.35\pm0.02$ in the skirts. The corresponding values for $G_L(G_{LR})$ were $0.71 \pm 0.08$ ($0.68 \pm 0.07$) and $0.16 \pm 0.03$ ($0.19 \pm 0.02$) for the central area and the skirts respectively.

*Example 8. In vitro cornea.* Polarization properties of *in vitro* samples (the isolated cornea in particular) can be calculated by using a polarimeter in transmission [36]. In a similar way, the Mueller matrix is obtained from 16 images of the sample corresponding to independent combinations generator–analyser (see [36] for further information). Figures 2(*a*) and (*b*) show the maps of $G_T$ and $G_L$ for an *in vitro* porcine cornea computed from the spatially resolved Mueller matrix (see [36] for further information). The difference $G_L$–$G_{LR}$ corresponds to figure 2(*c*). Mean values for the whole images are $0.73 \pm 0.04$, $0.46 \pm 0.05$ and $0.04 \pm 0.02$, respectively ($0.45 \pm 0.05$ for $G_{LR}$). The distribution for the polarization factor $d$ and the map for the difference $d - G_T$ (see the following section) are presented in figure 3.

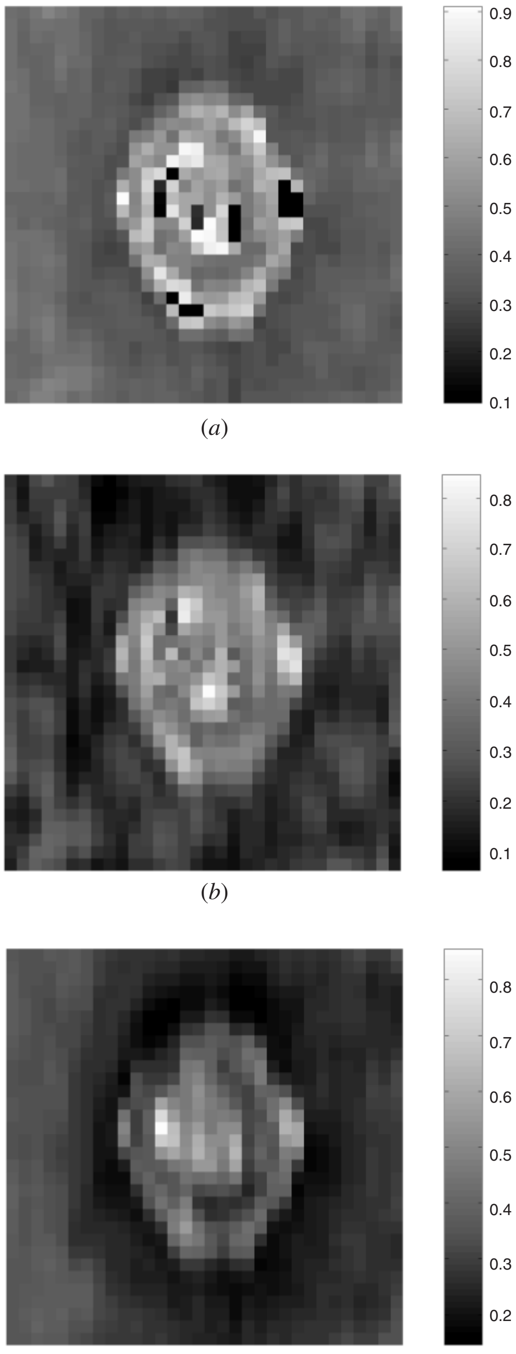


**Figure 1.** Spatially resolved $G_T$ (*a*), $G_L$ (*b*) and $G_{LR}$ (*c*) for the human retinal double-pass image. Each image subtends 29 min of arc of the visual field.

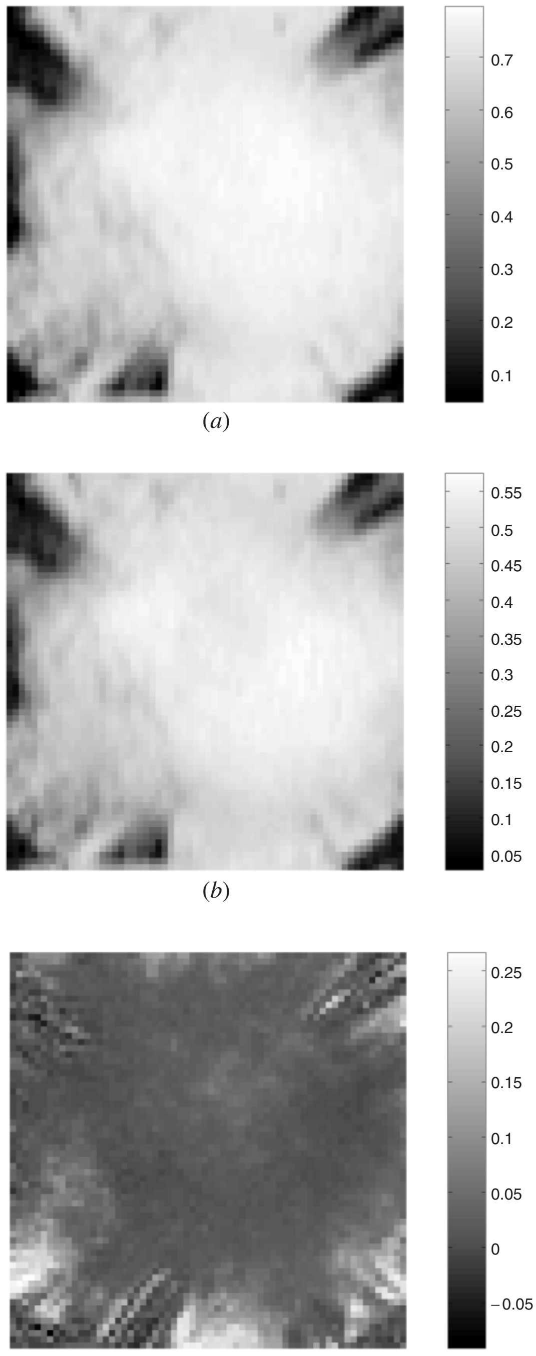


**Figure 2.** Maps for $G_T$ (*a*), $G_L$ (*b*) and $G_L$–$G_{LR}$ (*c*) corresponding to an *in vitro* porcine cornea. Each image has a full size of 10 mm. Note the difference in scale between (*a*) and (*b*).

## 4. Conclusions and discussion

Two new parameters, termed the direct and reverse indices of linear polarization ($G_L$ and $G_{LR}$) and computed from the elements of the Mueller matrix of a system, have been described. These indices measure how far a Mueller matrix is from an ideal linear polarizer.

The present parameters range from zero to one and verify that $G_L$ (and $G_{LR}$) $\leqslant G_T$. $G_T$ contains information on the depolarization properties of the system which is related to the DOP of the light emerging from the system with an (in general) partial elliptical polarization. Whereas $G_T = 1$ for all pure polarization elements such as polarizers, rotors and retarders, $G_L = 1$ only when the system acts as a total linear polarizer (see examples). On the other hand, when the emergent light is always depolarized the indices of linear polarization and the DOP are zero. However, $G_L$ (or $G_{LR}$) is also null when the emergent light is always circularly polarized (example 2).

Gil and Bernabeu [8] reported depolarization and polarization indices averaging the properties of the system in both directions. Although it is interesting to see what happens when averaging direct and reverse directions, much more information is gained by considering each direction separately. In our case, one parameter for each direction of the incident light has been described.

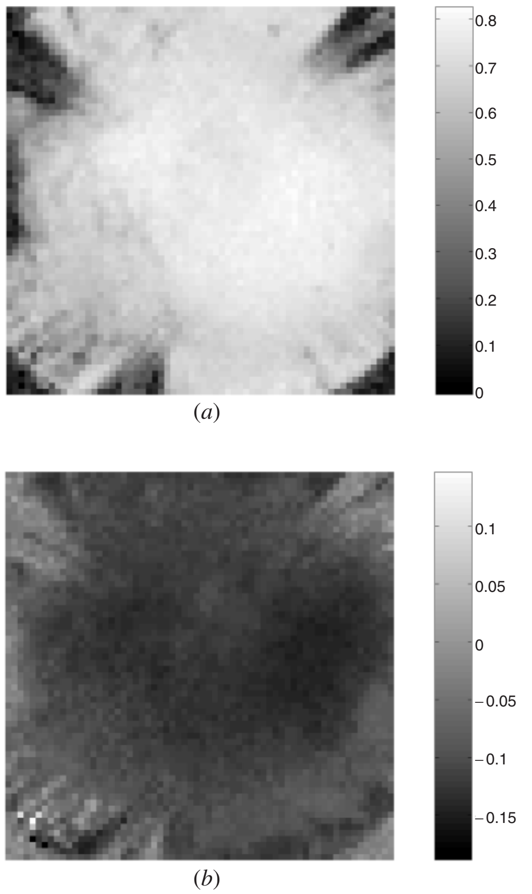


**Figure 3.** (*a*) Distribution for the factor of polarization, $d$ and (*b*) for the difference $d - G_T$ in the same *in vitro* cornea as in figure 2. Images have the same size as in the previous figure.

Chipman [37] studied the depolarization of Mueller matrices by mapping the resultant DOP for all possible polarized incident states. Previous experiments [38] had reported similar results of depolarization using an average DOP of the exiting beam (averaged over all possible totally polarized incident states) and the depolarization index defined in [8] (result of averaging over six Stokes vectors on the Poincaré sphere).

In this paper, values of $G_L$ (and $G_{LR}$) have been calculated for different ideal polarization elements and several combinations. When both diattenuation and depolarization are present in a system, the index depends on whether diattenuation occurs before or following depolarization (see, for instance, examples 2–4). For combinations of polarization elements in parallel, the indices of linear polarization for both directions of the incident light are the same (example 6).

The human eye is a paradigmatic example of interest where polarization changes, including depolarization, are present. As examples, the spatially resolved index of linear polarization for a human eye and an *in vitro* cornea have been shown and compared to the corresponding $G_T$. Maps for $G_T$, $G_L$ and $G_{LR}$ in figure 1 present a similar behaviour: parameters are larger for the central part of the images and reduce along the radius towards the skirts. Despite the presence of some noisy pixels in the images, in general, the indices of linear polarization are lower than $G_T$. Whereas in the central area $G_T$ is 12% higher than $G_L$ (or $G_{LR}$), in the skirts the difference is much larger (about twice as large). Differences between $G_L$ and $G_{LR}$ indicate that, in 'terms of polarization', the human eye is not totally symmetric ($m_{ij}^2 \neq m_{ji}^2$). This lack of symmetry might be due to the existence of ocular diattenuation (see examples) in addition to the intraocular scattering, depolarization effects and birefringent properties [25, 39, 40].

For the cornea alone (figure 2), the parameters are more uniform across the image. On average, $G_T$ is about 60% larger than $G_L$ (or $G_{LR}$). This contrasts with the result found for the whole eye where the difference is smaller than 20%. The reason could be the presence of more important effects of depolarization in the whole eye than in the cornea alone. Figure 2(*c*) shows that the corneal maps for direct and reverse indices are similar. This indicates that effects of diattenuation which 'breaks the symmetry in polarization' (when combined with depolarization and/or retardation) are negligible. In view of this, if the cornea acted as a linear retarder, the value for the indices of linear polarization would be $\sqrt{2}/2$ (table 1). However, depolarization is present for this sample ($G_T < 1$, figure 2(*a*)). Since $G_L = G_{LR} = d \cdot \sqrt{2}/2$ (example 5), the value of $d$ can be easily computed and is presented in figure 3(*a*). The average across the image is $0.70 \pm 0.03$, a value which is close to the mean obtained for $G_T$. This agrees with the fact that a partial depolarizer verifies that $G_T = d$ (table 1). Figure 3(*b*) shows that the difference between the parameters is small ($0.05 \pm 0.02$ on average).

Reducing the analysis to the $5 \times 5$ mm uniform central cornea (without the more external noisy areas), averaged values of $G_L$ and $G_{LR}$ are 0.54 and 0.53 respectively. With these data $d = 0.76$, a value similar to the mean for $G_T$ over the central area (0.77).

Howell [41] defined the DOLP for a system as:

$$G_{\mathrm{Howell}} = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}}$$

where $T_{\max}$ and $T_{\min}$, are the maximum and minimum intensity transmittances. As a function of the elements of the Mueller matrix these transmittances correspond to:

$$T_{\max} = m_{00} + \sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2}$$

$$T_{\max} = m_{00} - \sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2}$$

which implies that $G_{\mathrm{Howell}}$ is the diattenuation of the system, $D$, defined by Chipman [3] and also referred to as polarization sensitivity. Values of diattenuation are 0.15 and 0.04, for the eye (central part) and the cornea respectively.

Finally it is important to note the following [3]. If, while interacting with a system, a totally polarized state becomes partially polarized and then is polarized again (see, for instance, examples 3 and 4), depolarization is still present in the matrix ($0 < G_T < 1$) despite the fact that the emergent light beam is totally (linear) polarized. This fact is also present when calculating $G_L$.

To summarize, direct and reverse indices of linear polarization for an optical system have been presented. Only nine elements of the corresponding Mueller matrix are required. The information given by these indices is more complete than the diattenuation itself, because they include not only the polarization sensitivity, but also the effects of depolarization. Differences between direct and reverse indices are indicative of diattenuation. When this difference is negligible, only birefringent and depolarization properties might be present in the system. In such cases the parameter $G_T$ can be calculated without computing the full Mueller matrix.

## Acknowledgments

The author thanks P Artal for his assistance, helpful ideas and suggestions during the experiments about polarimetry in human eyes, J W Jaronski for recording the images corresponding to the *in vitro* corneas and J J Gil for comments on the first version of the manuscript.

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